Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series
Integral Calculus #11

Integration by Substitution – Strategic

Sometimes it simplifies an integral to make a substitution like $x = g(u)$ with $dx = g'(u)du$.

For example: There are several ways to simplify

$$\int \frac{1}{16 + x^2}\, dx$$

Let $x = 4u$ so that $dx = 4du$ AND $x^2 = 16\, u^2$

$$\int \frac{1}{16 + x^2}\, dx = \int \frac{1}{16}\frac{1}{1 + u^2}\, 4\, du = \frac{1}{4}\tan^{-1} u + C$$

Finally:

$$\int \frac{1}{16 + x^2}\, dx = \frac{1}{4}\tan^{-1}\frac{x}{4} + C$$

Example 2:

$$\int \frac{1}{\sqrt{9-x^2}}\,dx$$

Let $x = 3\sin u$ so $dx = 3\cos u\,du$

And $u = \sin^{-1}\frac{x}{3}$.

We will also use $\sin^2 u + \cos^2 u = 1$ in the form

$$\cos u = \sqrt{1-\sin^2 u}$$

Then

$$\int \frac{1}{\sqrt{9-x^2}}\,dx = \int \frac{3\cos u}{\sqrt{9-9\sin^2 u}}\,du$$

$$= \int \frac{3\cos u}{3\sqrt{1-\sin^2 u}}\,du = \int \frac{3\cos u}{3\cos u}\,du$$

$$= \int 1\,du = u + C = \sin^{-1}\frac{x}{3} + C$$